EXPERIMENTAL STRUCTURAL DYNAMICS

An Introduction to Experimental Methods of Characterizing Vibrating Structures

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Chapter I

BASIC VIBRATION CONCEPTS

1.1 Introduction

This text is about vibrating structures. The structures considered could be any of a broad range of engineered products, from TV sets, computers and other electronics products to cars, trucks, trains, aircraft and other vehicles. We could be talking about bridges or buildings. All of these products have the potential to fail in their product performance without proper engineering to avoid damage that could be caused by mechanical vibration. Aircraft are analyzed and tested to arrive at structural design characteristics that are successful in handling the aerodynamic loads encountered during flight. Vehicles are subject to vibration and noise originating from the engine, tires rolling over an irregular surface at high speed and turbulent air flow over the body. Vibration design characteristics are designed into the vehicle to avoid wear and fatigue failure of certain components and to provide a comfortable ride for the passenger.

For general arbitrary structures, the vibration process is very complicated, so complicated that one might expect the process impossible to comprehend. Impossible, except for the ability to analyze the most complex vibration motion as a superposition of relatively simple processes. It turns out that no matter how complicated the structure, and no matter how complicated the vibratory motion of the many parts of the vibrating structure, it is usually possible to separate the process into easily understood fundamental vibratory processes.

It is the goal of this text to first present the theory underlying the simple vibratory process, then develop the concepts allowing application of this understanding to the analysis of any complicated vibratory process for the most complex structure. There is one limitation in the level of structural complexity to be considered, however: The text will be concerned with linear structures. Vibration displacements will be small and stiffness characteristics will be fixed, independent of the amount of structural deformation.

1.2 Simple Harmonic Motion

A natural starting point is to study the motion of the simplest of structures in a natural state of vibration. Figure 1-1 depicts such a structure and the simple vibratory motion that results when a lumped mass sitting on a spring is made to vibrate freely. The mass is initially displaced upward from its equilibrium position on the spring. From this position it is released, accelerating downward under the pull of the stretched spring. The continuous motion of the mass is graphed with the solid curve in the figure. Instantaneous positions of the mass at key points in time are sketched. The mass is seen to oscillate, moving down until the upward force of the compressed spring brings the downward motion to a stop. Then the upward push of the compressed spring propels the mass upward until the cycle of oscillation is complete when the upward motion is stopped under the downward pull of the stretched spring. The cycle of motion is completed in one second in our example. From this point in time the mass will continue to oscillate in this fashion forever in the absence of any other influences, i.e, friction, human intervention, etc.

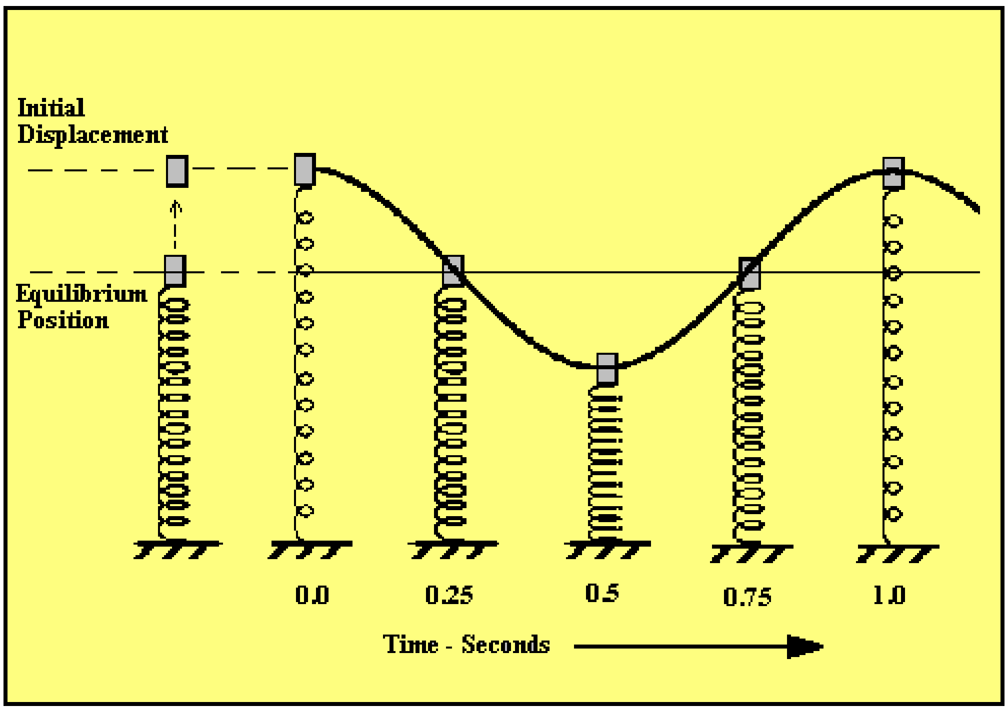

Figure 1-1. The vibration motion of a mass on a spring. After being displaced upward from the rest position and released, the mass oscillates in simple harmonic motion. The time period of oscillation is one second in our example.

Let's analyze the motion of the mass at each of the key points in time shown in Figure 1-1. The mass is released at the start of the process, 0.0 seconds. At the instant it is released, the stretched spring is exerting a pull causing an acceleration downward. However, the velocity is zero, the mass is at rest initially. So, at zero time we have the maximum positive displacement, zero velocity and maximum negative (downward) acceleration.

At 0.25 seconds the mass has returned to the equilibrium position, zero displacement. But now it is moving downward with maximum velocity. At the equilibrium position there is no force acting on the mass, so its acceleration is zero. Thus, the momentum of the mass will allow it to move right through the equilibrium position, after which its motion begins to be resisted as the spring is compressed.

The downward movement of the mass is stopped at 0.5 seconds after the continued compression of the spring overcomes the downward momentum, decelerating the motion to zero velocity. Now we have maximum negative displacement, zero velocity and maximum positive acceleration. The positive acceleration is due to the upward force of the compressed spring.

The continued upward thrust of the recoiling spring maintains an upward acceleration. The thrust diminishes as the spring uncoils, until at 0.75 seconds the spring has returned to the equilibrium position again, and the upward thrust and acceleration are zero. So now we have zero displacement, maximum positive velocity and zero acceleration.

Finally, one cycle of vibration is complete at 1.0 second as the upward momentum carries the mass upward to the maximum displacement position where it is momentarily brought to rest again by the downward pull of the stretched spring. The final conditions are the same as at the start of the process, when the mass was first released: Maximum positive displacement, zero velocity and maximum downward (negative) acceleration.

Figure 1-1 illustrates one way of initiating free vibration in a simple mass-spring system. The mass is given an initial displacement and released. Now, consider a way of initiating vibration without displacing the mass initially. Figure 1-2 depicts the initiation of vibration by striking the mass with a hammer, almost instantaneously imparting an initial maximum upward velocity to the mass, which is hanging on a spring, initially at the equilibrium position. For all practical purposes we can consider the time duration that the hammer is in contact with the mass to be infinitessimally small. The measure of smallness is relative to the time period of one cycle of oscillation.

The following conditions summarize the motion of Figure 1-2. At 0.0 seconds (initial condition) the displacement is zero, velocity is a positive maximum and acceleration is zero (after an infinitessimal time increment during which the impact transient accelerates the mass to the initial velocity). At 0.25 seconds displacement is at the positive peak, velocity is zero and acceleration is a negative maximum. At 0.5 seconds displacement is zero (returned to the equilibrium position), velocity is a negative maximum and acceleration is zero. At 0.75 seconds displacement is maximum negative, velocity is zero and acceleration is maximum positive. The mass returns to the equilibrium position and has the original conditions of zero displacement, maximum positive velocity and zero acceleration.

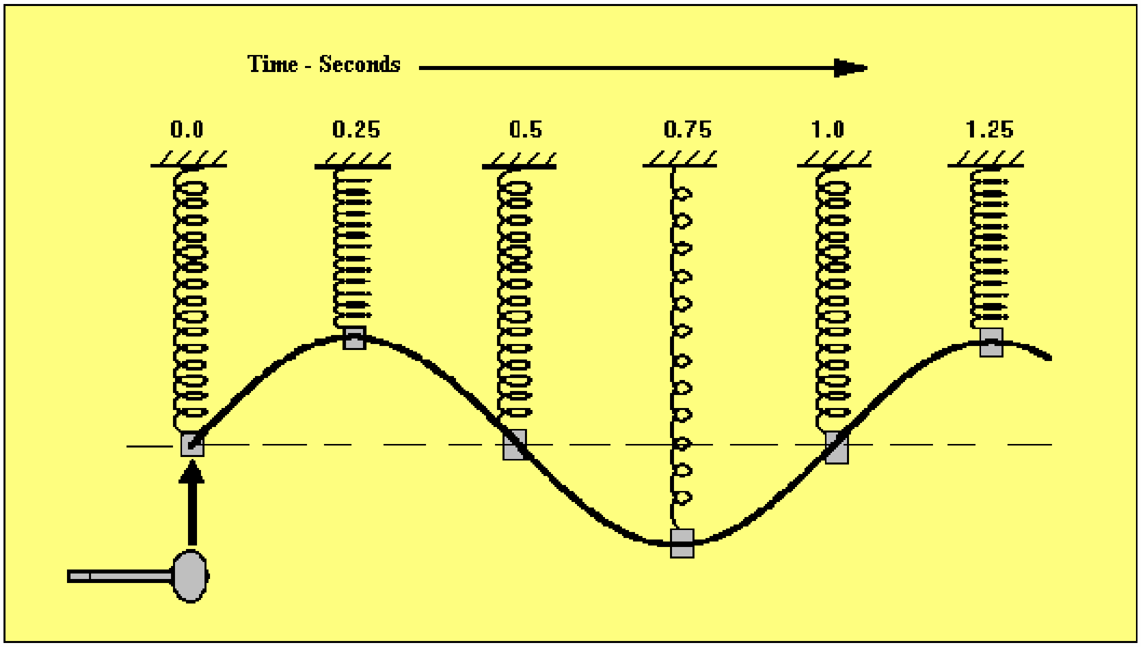

Figure 1-2. The hammer impact is another way of exciting a mass and spring into free vibration. The pulse duration is so short compared to the time period of one vibration cycle that, for all practical purposes, the process begins with a condition of maximum velocity.

Throughout this text the motion of masses or points on a vibrating structure will often be described by plotting either the displacement, velocity or acceleration versus time. Figure 1-3 is a plot of displacement versus time for the mass bouncing on the spring as a result of the hammer impact depicted in Figure 1-2. The Figure 1-3 plot follows the solid curve that traces the Figure 1-2 mass motion, extending the plot over several cycles of vibration. The vibration is plotted over a period of six seconds.

Two parameters are used in conjunction with the trigonometric sine function to describe the oscillatory motion: 1) Amplitude and 2) Frequency. The amplitude in Figure 1-3 refers to the maximum or peak displacement, which is 1.0 inch in this example. The frequency refers to the rate of oscillations, given in units called Hertz (Hz), or number of cycles per second. The oscillations in Figure 1-3 occur at a rate of one completed cycle of motion each second, or 1.0 Hz. The Figure1- 3 plot is also described by the algebraic formula:

$$Y(t) = Y_{max}\sin(2\pi\nu t) \qquad (1\text{-}1)$$

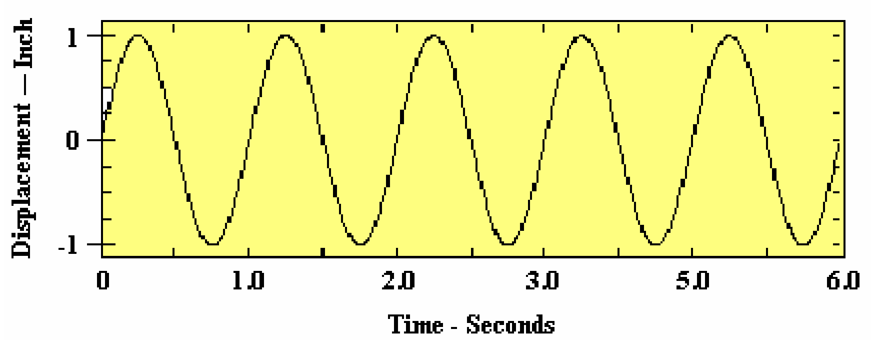

Figure 1-3. Graph of displacement versus time for the vibration process illustrated in Figure 1-1.

where Y is the instantaneous displacement at any time, t, Y_{max} is the amplitude and ν (greek letter nu) is the frequency.

The motion represented by Figure 1-2 and Figure 1-3 is known as simple harmonic motion or sinusoidal motion. The sine function used in Equation (1-1) represents the displacement versus time trigonometric relation. Figure 1-4 helps clarify the trigonometric relation.

The left side of Figure 1-4 represents the motion of a point as it moves counter clockwise around the circumference of a circle whose radius is Y_{max}. The point progresses through a sequence of positions spaced 45 degrees apart. These points are numbered 1 through 8. Imagine that the point is continually moving with this circular motion and that it takes one second to move all the way around the circle once. Completing one trip around the circle will be referred to as one cycle of motion. The frequency or rate of moving around the circle is then one cycle per second.

Now, focus attention on just the vertical displacement of the point at each numbered position. The vertical line through the center of the circle in the figure is labeled as the Y axis, and the vertical displacement will be referred to as the value of Y. It is the projected position of the point onto the Y axis that actually represents the up and down vibration of the mass. The motion of the point around the circle is being used to assist the understanding of the way that the trigonometric sine function along with amplitude and phase angle represent the vertical oscillation.

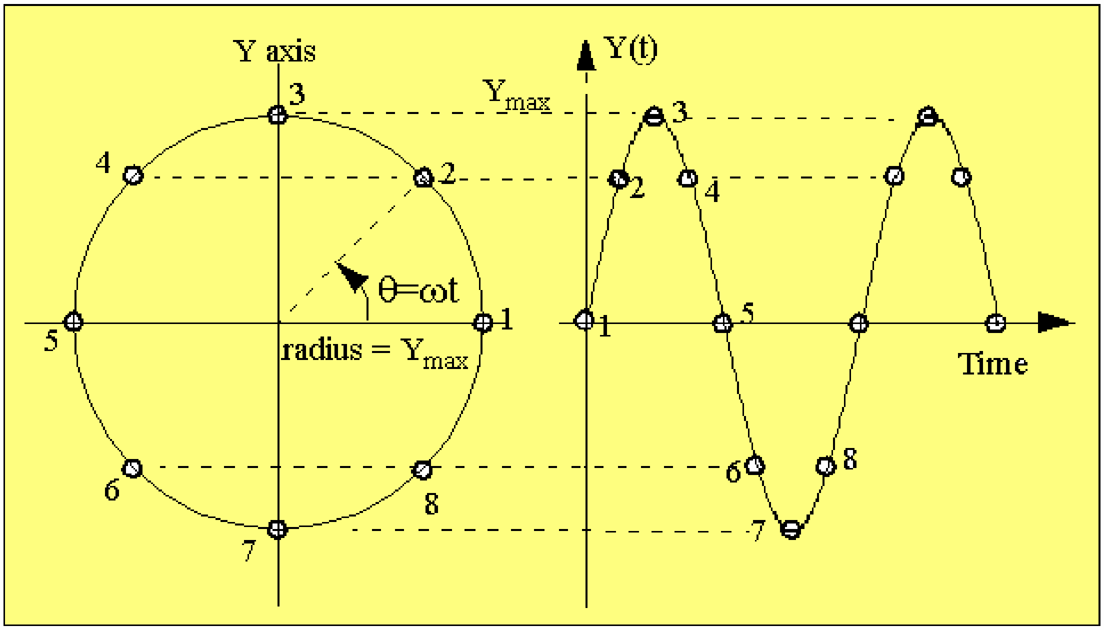

Figure 1-4. A sequence of positions moving counter clockwise around a circle through angle, theta (θ), is matched to the corresponding positions on a graph of displacement versus time for a simple harmonic oscillator.

Figure 1-5 constructs a right triangle with angle, θ (greek letter theta). Recall from trigonometry that the sine of the angle, θ, is defined as the ratio of the side opposite the angle (labeled as Y) divided by the hypotenuse (labeled as radius, Ymax).

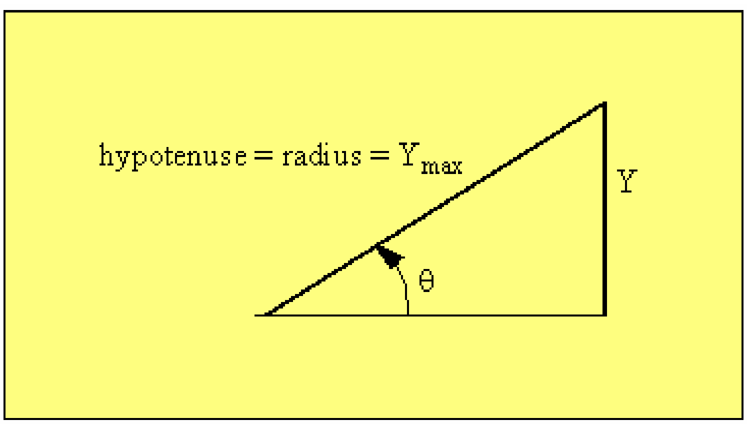

Figure 1-5. A right triangle with defining parameters for the trigonometric sine definition.

This is expressed by the equation:

$$\sin(\theta) = \frac{Y}{\text{radius}} \tag{1-2}$$

Or, using the value, Y_{max}, for the hypotenuse or radius,

$$\sin(\theta) = \frac{Y}{Y_{max}} \tag{1-3}$$

It is seen from equation (1-3) that the Y displacement value for any angle, θ, and peak displacement or circle radius, Y_{max}, is:

$$Y = Y_{max}\sin(\theta) \tag{1-4}$$

Unless otherwise indicated, this text will always use the unit, radian, when referring to a variable angle, θ. One radian of angle is that angle swept out when moving counter clockwise along the circumference of a circle such that the arc length along the circumference is equal to the radius of the circle. Since the circumference of a circle is known to be equal to 2π times the radius, then an angle of 2π radians would be swept out by moving around the complete circumference of the circle. This also implies that 2π radians correspond to 360 degrees. And since moving around the complete circumference of the circle, i.e., through 360 degrees or 2π radians, corresponds to one cycle of motion in Figure 1-4, it is seen that

$$1 \text{ cycle} = 2\pi \text{ radians} \tag{1-5}$$

A special symbol, ω (greek letter omega), will be used to represent the rate of change of angle in radians with passage of time. Therefore, the angular position, θ, of the point moving around the circle circumference can be represented at any time, t, as

$$\theta = \omega t \tag{1-6}$$

Now, a formula for the vertical position of the point along the Y axis for any time, t, can be expressed:

$$Y(t) = Y_{max}\sin(\omega t) \tag{1-7}$$

Since there are 2π radians per cycle (equation (1-5)), and frequency in Hz, ν, is cycles per second, the relationship between frequency in radians per second and frequency in Hz is

$$\omega = 2\pi\nu \tag{1-8}$$

Equation (1-7) can now be written using units of Hz (Hertz, cycles per sec), ν:

$$Y(t) = Y_{max}\sin(2\pi\nu t) \tag{1-9}$$

The application of the trigonometric sine function of equation (1-9) is the basis for referring to the curves of Figure 1-3 and right side of Figure 1-4 as sine waves. Matching the sequence of numbered positions between the circle on the left side to the sine wave on the right side of Figure 1-4, makes clear the use of the trigonometric sine function in the mathematical description of a sine wave.

The sine waveform is fundamental to the understanding of structural vibrations. It will be seen that even when the motion of a vibrating structure is complex, characterized by irregular movements, that motion can still be understood as a superposition of many different sine waves, each having different amplitudes and frequencies.

1.3 Velocity Sine and Cosine Wave Forms

When describing the oscillating mass of Figure 1-2 we took note of the velocity at key points of the oscillation cycle. The velocity was a maximum at the start of the process, i.e., at the first instant the mass was set in motion by the hammer impact. Then the velocity slowed to zero at 0.25 seconds under the breaking reaction force of the stretching spring (the peak displacement position), changed to a negative maximum at 0.5 seconds (the equilibrium zero displacement position), decreased to zero velocity at 0.75 seconds (the maximum negative displacement position), and finally the completion of one cycle of oscillation at 1.0 second brought the mass back to the initial displaced position with the original maximum positive velocity. The time history of velocity is represented by the plot of velocity versus time in Figure 1-6.

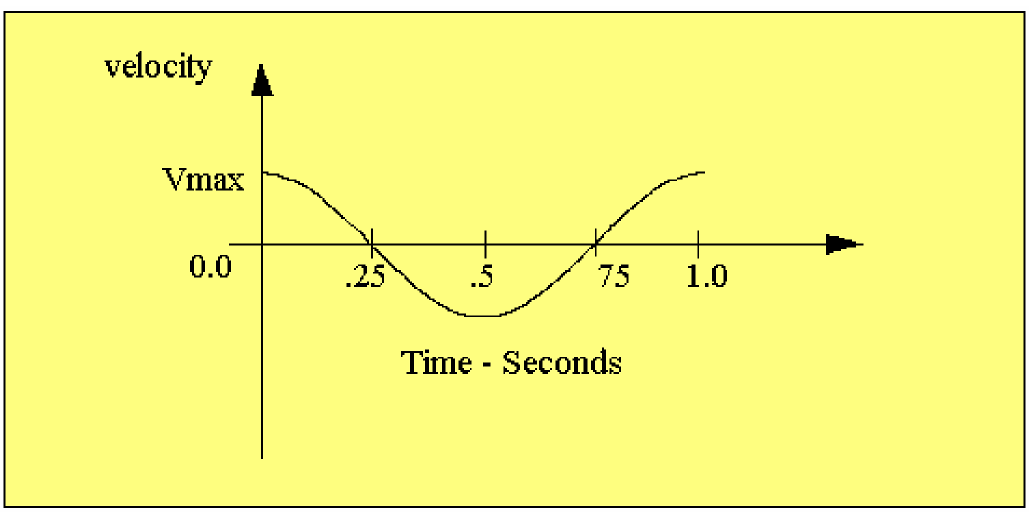

Figure 1-6. Velocity versus time for Figure 1-2 vibration. The vibration when a hammer imparts an initial maximum velocity.

A sine function may be used to represent the velocity versus time plot of Figure 1-6, but a phase angle must be added to the argument to account for the way velocity starts at t=0 with a maximum value. The phase of the velocity is shifted by 90^o, or $\pi/2$ radians. The formula for the velocity, V(t), sine wave with phase shift is

$$V(t) = V_{max}\sin(\omega t + \pi/2) \tag{1-10}$$

The 90^o phase shift for the circular motion of a point and the related phase shifted sine wave is shown in Figure 1-7. Note that the point motion begins at time, t = 0, at the angle, $\theta = \pi/2$ (90 degrees), rather than beginning at the angle, $\theta = 0$.

Of course this wave form is also known as a cosine function of time. The algebraic relationship between the cosine and sine functions is:

$$\cos(\theta) = \sin(\theta + \pi/2) \tag{1-11}$$

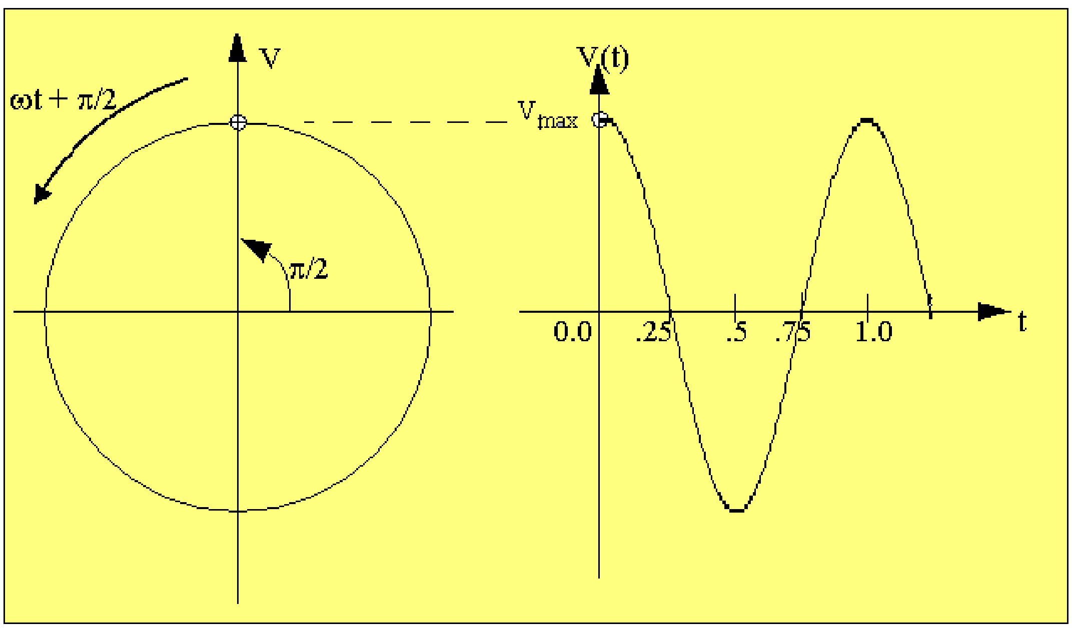

Figure 1-7. Using a phase shift to allow a trigonometric sine function to describe the velocity wave form, which starts at a maximum value, V_{max}.

Using the cosine definition avoids the need for a phase shift in the algebraic expression:

$$V(t) = V_{max}\cos(\omega t) \tag{1-12}$$

Or, using frequency, ν, in Hz, equation (1-12) is written

$$V(t) = V_{max}\cos(2\pi\nu t) \qquad (1\text{-}13)$$

The displacement sine wave and velocity sine wave with phase shift (velocity cosine wave) are overlayed for comparison in Figure 1-8. The displacement sine function starts at a value of zero and the velocity phase shifted function starts at a positive maximum value. The wave forms are plotted over a time interval of three seconds and have a frequency of one Hz.

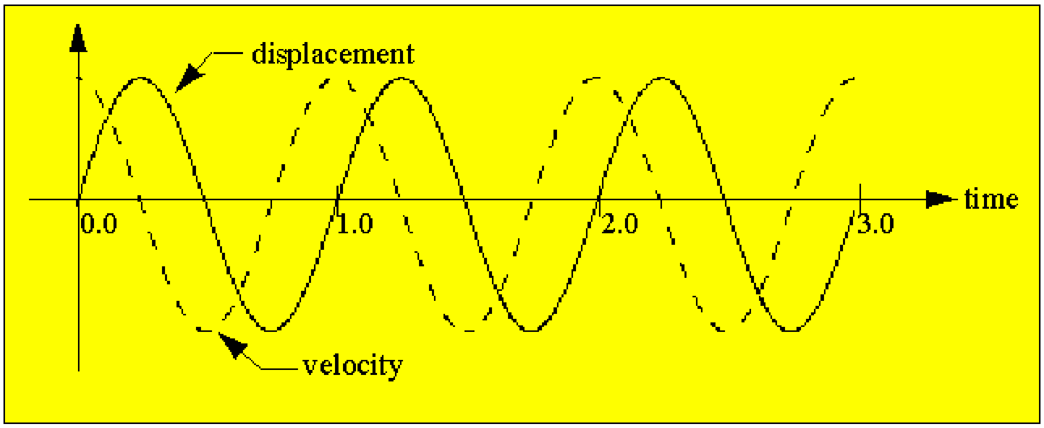

Figure 1-8. Comparison of displacement and velocity wave forms for the simple harmonic motion of the mass bouncing on a spring in Figure 1-2.

A triangle is sketched in Figure 1-9 to show the parameters used in trigonometry for defining a cosine function. The cosine is defined as the ratio of the side, V, adjacent to the angle, θ, divided by the hypotenuse, V_{max} . Figure 1-10 compares the use of the trigonometric sine and cosine definitions for describing the velocity wave form as either a phase shifted sine wave or a cosine wave form with zero phase angle.

The definition of cosine for the Figure 1-9 sketch is:

$$\cos(\theta) = V/V_{max} \qquad (1\text{-}14)$$

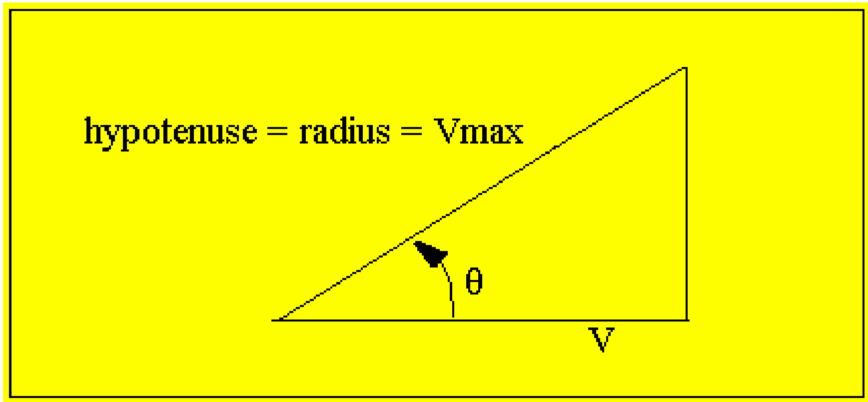

Figure 1-9. Parameters used in defining cosine(θ): the ratio of the side adjacent to the angle (velocity, V) divided by the hypotenuse, V_{max}.

The value of V may then be written as:

$$V = V_{max}\cos(\theta) \qquad (1\text{-}15)$$

The formula for the cosine description of the velocity wave form plotted in the lower left part of Figure 1-10 was written as equation (1-12) and is repeated here as equation (1-16):

$$V(t) = V_{max}\cos(\omega t) \qquad (1\text{-}16)$$

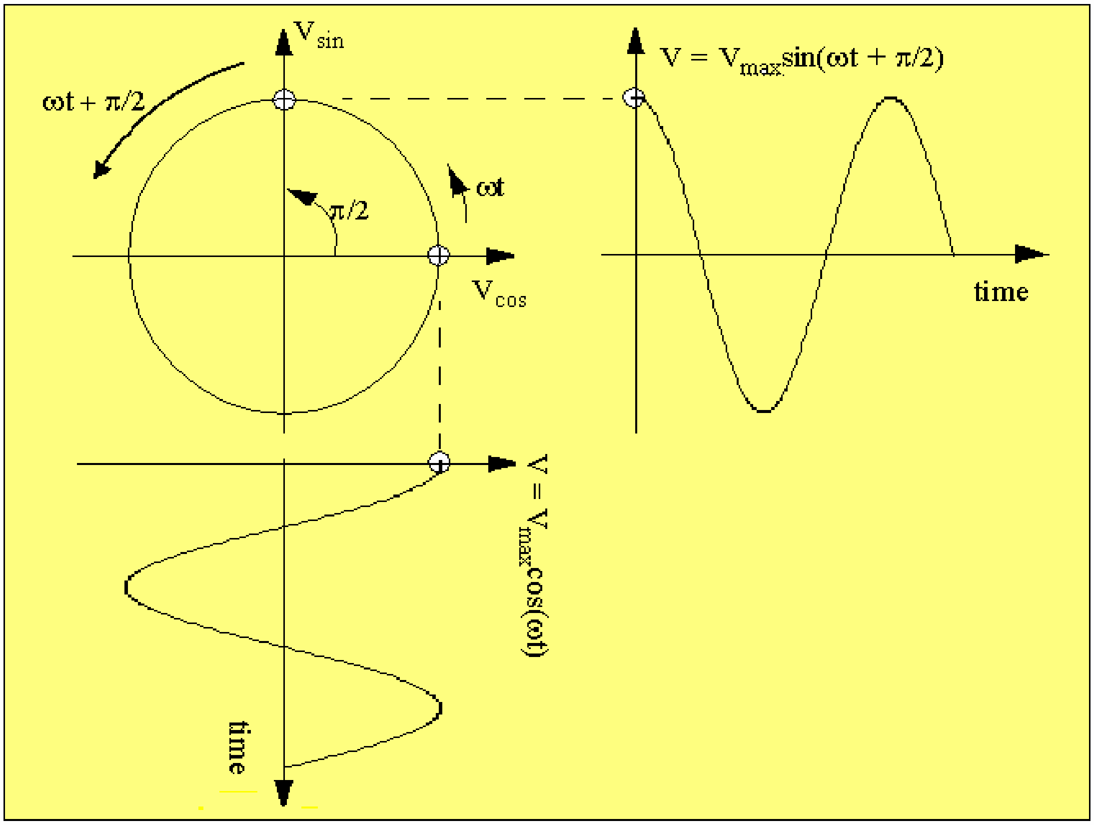

Figure 1-10. A comparison of two ways of representing the velocity wave form. Upper right shows the phase shifted sine wave. The motion of the point on the circle begins at the π/2 phase position. The lower left plot is the cosine wave form. The point on the circle starts at the zero phase position.

1.4 Acceleration Sine Wave Form

The acceleration of the vibrating mass was noted at the key positions shown in Figure 1-2. At time equal to zero the acceleration was zero. When the displacement at 0.25 seconds was a positive maximum, the acceleration was maximum negative. At the 0.5 second equilibrium position the acceleration was zero. Acceleration was a positive maximum at 0.75 seconds when the position was a negative extreme. At the completion of one cycle the acceleration was again zero. The acceleration function of time, A(t), is a sine function, but the sign is the opposite of the displacement. The formula is

$$A(t) = -A_{max}\sin(\omega t) \quad (1\text{-}17)$$

The acceleration versus time wave form for Figure 1-2 is plotted in Figure 1-11.

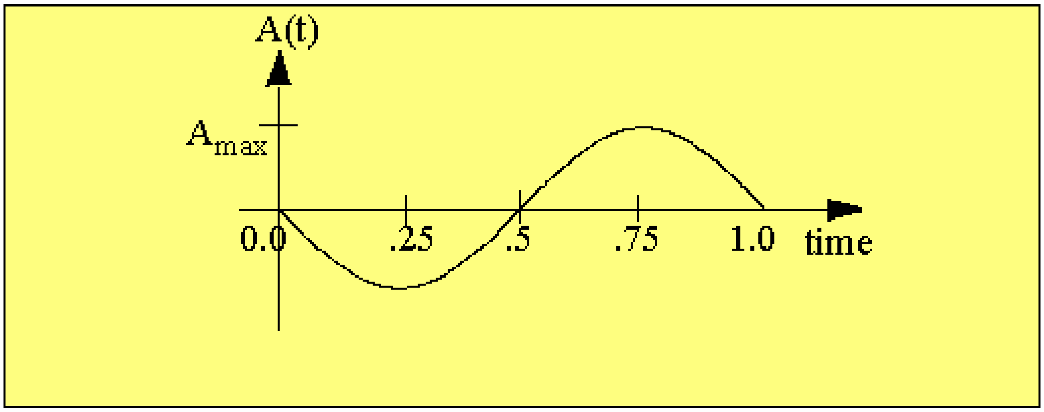

Figure 1-11. Acceleration versus time for the oscillatory motion of the mass bouncing on the spring in Figure 1-2. The acceleration is described by a sine function with opposite sign from the displacement (180 degrees out of phase with displacement).

The relationship between displacement, velocity and acceleration is summarized in Figure 1-12.

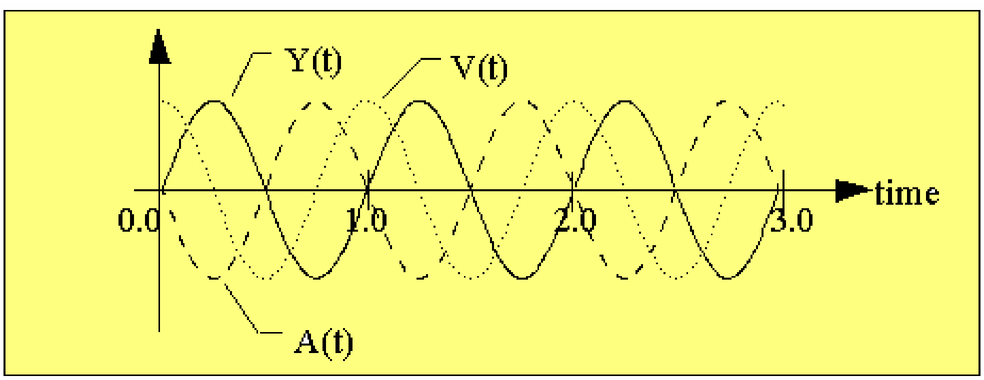

Figure 1-12. Comparison of wave forms for displacement, Y(t), velocity, V(t), and acceleration, A(t), for simple harmonic motion.

1.5 Velocity and Acceleration Amplitudes

We've been skirting the details of the actual velocity and acceleration amplitudes associated with sinusoidal displacement wave forms. The overlays in Figure 1-12 were not intended to properly reflect amplitude relationships. The velocity and acceleration formulas will now be developed using calculus derivatives, beginning with a repetition of the displacement sine wave formula of equation (1-7),

$$Y(t) = Y_{max}\sin(\omega t) \quad (1\text{-}18)$$

where Y_{max} is the amplitude (maximum displacement) value.

The velocity, V(t), is the first derivative of displacement with respect to time, resulting in

$$V(t) = \omega Y_{max}\cos(\omega t) \quad (1\text{-}19)$$

Or, using frequency, ν, in Hertz,

$$V(t) = 2\pi\nu Y_{max}\cos(2\pi\nu t) \quad (1\text{-}20)$$

Comparing this result, equation (1-20), with equation (1-13) the amplitude or maximum value of the velocity cosine wave form is related to the displacement amplitude and frequency as

$$V_{max} = 2\pi\nu Y_{max} \quad (1\text{-}21)$$

The acceleration function of time, A(t), results from taking the second derivative of displacement or the first derivative of velocity. Taking the first derivative of the velocity function of equation (1-19) gives

$$A(t) = -\omega^2 Y_{max}\sin(\omega t) \quad (1\text{-}22)$$

or, again using frequency, ν, in Hertz,

$$A(t) = -4\pi^2\nu^2 Y_{max}\sin(2\pi\nu t) \quad (1\text{-}23)$$

Comparing equation (1-23) to equation (1-17) shows the acceleration sine wave amplitude or maximum value, A_{max}, to be

$$A_{max} = 4\pi^2\nu^2 Y_{max} \quad (1\text{-}24)$$

Notice the squared frequency relationship between displacement and acceleration. This accounts for the often observed vibration lab phenomena in which a large displacement at a very low frequency is accompanied by a small acceleration level. At very high frequencies and small displacements, perhaps microinches, the acceleration can be quite high.

The units used in equation (1-20) and equation (1-23) for velocity and acceleration depend on the units being used for displacement. If displacement is expressed in units of inches, then velocity is in units of inches/sec and acceleration is in units of inches/sec/sec.

Acceleration is usually the parameter measured for characterizing the vibration of a structure. This is because of the difficulty of performing accurate measurements of displacement or velocity over a wide frequency range. Also, test article measurment locations are too often inaccessable to displacement and velocity measurement devices. Acceleration, on the other hand, is easily measured using devices called accelerometers. Test labs usually calibrate their accelerometer instrumentation channels to provide data in units of G's. An acceleration wave form, G(t), in units of G's is obtained by forming the ratio of acceleration, A(t), to the constant acceleration of gravity, g_0:

$$G(t) = \frac{1}{g_0} A(t) \quad (1\text{-}25)$$

where g_0 = 386.4 in/sec/sec. Writing equation (1-17) again, using G's and frequency, ν, in Hertz,

$$G(t) = -\frac{A_{max}}{386.4} \sin(2\pi \nu t) \quad (1\text{-}26)$$

Or,

$$G(t) = -G_{max} \sin(2\pi \nu t) \quad (1\text{-}27)$$